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Simulations of waves in sunspots

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Abstract. Magnetic field modifies the properties of waves in a complex way. Significant advances has been made recently in our understanding of the physics of waves in solar active regions with the help of analytical theories, numerical simulations, as well as hi-resolution observations. In this contribution we review the current ideas in the field, with the emphasis on theoretical models of waves in sunspots.

1. Motivation and questions

Photospheres and chromospheres of sunspots are the regions where different physical agents come into play with almost equal weight. The restoring forces in the wave equation, such as magnetic Lorentz force, gas pressure gradient and buoyancy, are of the same order of magnitude, making arbitrary the division into pure wave modes and complicating theoretical models. A recent summary on the observational properties of sunspot oscillations together with some theoretical issues is presented by Bogdan & Judge (2006). The most recent observational results suggest that wave phenomena at different layers of the sunspots umbrae and penumbrae are related with each other (see *e.g.* Marco et al. 1996; Maltby et al. 1999, 2001; Brynildsen et al. 2000, 2002; Christopoulou et al. 2000, 2001; Rouppe van der Voort et al. 2003; Tziotziou et al. 2006; Bloomfield et al. 2007a). The interpretation of the observational material defines several groups of questions concerning the sunspot wave physics that can be summarized in the following way:

- What drives the waves observed in sunspots? Are they externally driven by the quiet Sun p -modes? Are there sources of oscillations inside the umbra due to weak convection?
- How can the oscillations observed at different sunspot layers be interpreted in terms of MHD waves? What are the relationships between photospheric and chromospheric oscillations? What causes the complex spatial pattern of oscillations such as chromospheric umbral flashes, penumbral waves, spatial coherency of waves over the umbra, etc.?
- Why is the wave power suppressed in the umbral photosphere compared to the quiet Sun? Why is it enhanced in the chromosphere?
- What mechanisms produce the change of the dominating frequency of waves in the umbra from 3 mHz in the photosphere to 5–6 mHz in the

chromosphere? What are the reasons for the spatial power distribution at different frequency intervals observed in the umbra and penumbra?

- Are there observational evidences of the mode transformation in sunspots?
- What is the source of helioseismological velocity signal detected in active regions? Are they due to fast, slow MHD waves? What are the consequences of the strong magnetic field of sunspots onto helioseismology measurements and inversions of sub-photospheric structure of sunspots?

Some of these questions, like the interpretation of the wave modes or change of the frequency with height, received larger attention in the literature in the past and can be considered answered to a smaller or larger extent. Some others, like those concerning local helioseismology in active regions, only received attention recently and no unique answer has been found yet. In the rest of paper we will describe the existing models and the theoretical progress made in understanding sunspot oscillations.

2. Excitation of oscillations in sunspots

It is generally accepted that the quiet-Sun oscillations are stochastically excited by solar convection at the top part of the convection zone (Goldreich & Kumar 1990; Nordlund & Stein 2001). The frequency spectrum and the temporal behaviour of waves in the umbra is very similar to the quiet Sun, except for reduced power (*e.g.* Penn & LaBonte 1993). It is systematically found in observations that more wave power travels toward the umbra than leaves the umbra. One of the natural explanation of this effect is to assume that the sunspot waves are driven by external p -modes, modified inside the magnetized sunspot atmosphere.

A number of theoretical investigations has been performed assuming the incoming p -mode waves incident on a sunspot-like magnetic field concentration (*e.g.* Cally et al. 1994; Cally & Bogdan 1997; Rosenthal & Julien 2000) and studying the wave transformation, “absorption” and scattering by sunspots. These studies have shown that both amplitudes and phases of the incoming waves are modified after passing through the magnetic field regions. In particular, the surface amplitudes of the p -modes were found to be reduced due to the mode transformation (Sect. 5).

Despite the presence of strong magnetic field in sunspot’s umbra and penumbra, the models of magnetoconvection indicate that these environments can be convective (Lee 1993; Weiss et al. 1990; Cattaneo et al. 2003; Schüssler & Vögler 2006; Spruit & Scharmer 2006). The narrow nearly field-free upflowing umbral dots adjacent by downflows are the result of convection in a strong magnetic field. Recent observations seem to confirm this idea (see *e.g.* Bharti et al. 2007; Watanabe et al. 2008). Thus, some waves can be generated inside the sunspot umbra as well. Jacoutot et al. (2008) performed numerical simulations of magnetoconvection and studied the spectra of generated waves as a function of magnetic flux in the model. Apart from the power suppression in regions with enhanced magnetic field, these simulations suggest an increase of high-frequency power (above 5 mHz) for intermediate magnetic field strengths (of the order of

300–600 G) caused by changes of the spatial-temporal spectrum of turbulent convection in a magnetic field.

It can be concluded that both external driving and in-situ excitation by convection are able explain the observed properties (*e.g.* power reduction) of waves in sunspots. A qualitative estimation of the effect of the reduced wave excitation in sunspots and the direct comparison to observations was reported by Parchevsky & Kosovichev (2007). The authors performed hydrodynamical simulations of waves generated by random sources, with the strength of the sources reduced to zero in the sunspot umbra. They obtained that, even though no waves were excited in the umbra, the velocity power measured there was about twice larger than actually observed. Thus, the waves detected in sunspots must be a mixture of external p -modes (and other wave types produced by the mode conversion) and MHD waves generated directly inside the sunspot by the weak convection. The relative contribution of the different effects is to be determined.

3. Interpretation of oscillations in terms of MHD waves

Historically, the sunspot oscillations were divided into 5-min photospheric oscillations, 3-min chromospheric oscillations and running penumbral waves (Bogdan & Judge 2006). Recent observations show that these oscillations can be a manifestation of the same physical phenomena (Maltby et al. 1999, 2001; Brynildsen et al. 2000, 2002; Christopoulou et al. 2000, 2001; Rouppe van der Voort et al. 2003; Centeno et al. 2006; Tziotziou et al. 2006, 2007). Chromospheric observations show umbral flashes followed by a smooth continuous quasi-circular expansion of the visible perturbation pattern from the umbra through the penumbra toward the edge of sunspot. At the same time, the frequency of chromospheric oscillations decreases from 5 to 3 mHz from the umbra to the penumbra (Tziotziou et al. 2007). The waves in the umbra propagate from the photosphere to the chromosphere along the magnetic field lines forming shocks of about 10 km/sec peak-to-peak amplitude at heights of formation of He I 10380 Å line (Lites 1986; López Ariste et al. 2001; Rouppe van der Voort et al. 2003; Centeno et al. 2006).

The characteristic speeds of waves change many orders of magnitude along the photosphere and chromosphere of sunspots. At some height waves propagate through the region where the sound and the Alfvén speed are equal ($c_S = v_A$). In this region mode transformation and coupling of different wave phenomena occurs. In addition, the magnetic field and thermodynamic variables show important gradients both in horizontal and vertical directions. All these ingredients make realistic modeling of sunspot waves a rather difficult task only accessible via numerical simulations. Theoretical bases of the MHD wave propagation in a stratified atmosphere with a constant magnetic field were developed by, *e.g.*, Ferraro & Plumpton (1958); Osterbrock (1961); Zhugzhda & Dzhililov (1982, 1984); Evans & Roberts (1990). One of the first numerical simulations of waves in non-trivial magneto-atmospheres with applications to solar photosphere and chromosphere was performed by Rosenthal et al. (2002); Bogdan et al. (2003). Similar calculations for conditions appropriate for sunspots rather than flux tubes were carried out by Khomenko & Collados (2006). The waves were generated by a photospheric pulse located inside the magnetic field region and propa-

gated through the layer where $c_S = v_A$ on their way from the photosphere to the chromosphere. Rosenthal et al. (2002) and Bogdan et al. (2003) pointed out an important role of the magnetic canopy defined as the region where the plasma β ($\beta = 8\pi p/B^2 = (\gamma/2)c_S^2/v_A^2$) is close to unity. According to their results, the properties of waves observed in the atmosphere depend sensitively on the location and orientation of the magnetic canopy. The mode transformation occurs in the canopy region and several wave types can be present simultaneously there traveling in different directions. However, the fast and the slow wave modes decouple effectively away from the transformation layer. At heights above $\beta = 1$ the refraction and reflection of the fast (magnetic) mode occur (Rosenthal et al. 2002; Khomenko & Collados 2006). The slow mode (either produced by the driver or after the mode transformation) is always guided along the magnetic field lines, except at the region $\beta \approx 1$ where it deviates from this direction by maximum 27 degrees (Osterbrock 1961). These and other theoretical investigations demonstrated that the explanation of the observed oscillations in magnetic structures in terms of MHD waves requires the knowledge of the location of the $\beta = 1$ level, as well as the inclination of the magnetic field lines.

Mathew et al. (2004) used spectropolarimetric observations in Fe I 1.56 μm lines to calculate maps of plasma β for the observed sunspot. The plasma beta at heights of the continuum formation of radiation at 1.56 μm was found to be less than 1 in most of the umbra and inner penumbra reaching values above 1.5 in the outer penumbra. Most photospheric lines used typically in observations (like Ni I 6768 Å used by SOHO/MDI) are formed even higher in the atmosphere. It indicates that oscillations observed in sunspot umbra should be due to the low β MHD waves. The situation is more complex in the penumbra where the plasma β should be around unity at the typical heights of formation of photospheric spectral lines.

From observations, there is no clear conclusions on what wave types are detected in sunspots. The analysis of magnetic field oscillations in a sunspot obtained from spectropolarimetric observations of Fe I 1.56 μm lines allowed Khomenko et al. (2003) to conclude that the observed waves are a mixture of fast and slow low- β waves and the contribution of the slow (acoustic-like) wave is the dominant in the sunspot umbra. The slow (acoustic-like) wave propagation along the magnetic field lines from the photosphere to the chromosphere in the sunspot umbra was reported from spectropolarimetric observations by Centeno et al. (2006). In the penumbra, Bloomfield et al. (2007b) obtained that the propagation speeds of waves in the deep photosphere most closely resemble the fast-mode speeds of the modified p -modes, *i.e.* high- β acoustic-like waves propagating toward the sunspot at an angle ~ 50 degrees to the vertical. In a subsequent paper, Bloomfield et al. (2007a) used a time series of photospheric Si I and chromospheric He I spectra at 1.08 μm and concluded that the running penumbral waves are a visible pattern of low- β slow mode waves propagating and expanding their wavefront along the inclined magnetic field lines in the penumbra. Similar interpretation of the running penumbral waves was also suggested by Bogdan & Judge (2006). Anticipating the conclusions from Sect. 6 of this paper, theoretical modeling of local helioseismology signals in sunspot regions seem to suggest that they are rather due to high- β fast mode waves (modified

p -modes) (Khomenko et al. 2008). Given these diverse conclusions, more work is required to unify the picture of MHD wave modes observed in sunspots.

4. Frequency distribution

The power of velocity and intensity oscillations is distributed in a complex way over the active regions (Hindman & Brown 1998; Jain & Haber 2002; Tziotziou et al. 2007; Nagashima et al. 2007; Mathew 2008). In the photosphere, the power of oscillations is generally suppressed at all frequencies, both in the umbra and in the penumbra, except for the bright ring at the umbra-penumbra boundary (Mathew 2008). Acoustic halos are found at the edges of active regions depending magnetic field strength (Braun et al. 1992; Brown et al. 1992; Hindman & Brown 1998) being most prominent at high frequencies above 5 mHz. In the chromosphere, the power is suppressed at low frequencies (below 2.5 mHz) in the umbra but then is enhanced in the high frequencies (above 3.5 mHz) (*e.g.* Nagashima et al. 2007). The most prominent are 5–6 mHz oscillations observed in the umbral chromosphere. The running penumbral waves in the chromosphere have lower frequency around 3 mHz (Tziotziou et al. 2007).

Assuming the waves observed in the umbral photosphere and chromosphere are slow low- β acoustic-like waves propagating along nearly vertical magnetic field lines, the mechanism of the frequency variation with height of these waves should be similar to that of the ordinary acoustic waves in the non-magnetic Sun. Fleck & Schmitz (1991) have demonstrated that the change of frequency with height from 3 to 5–6 mHz is due to a resonant excitation at the atmospheric cut-off frequency, occurring even for linear waves in an isothermal atmosphere (see also Kalkofen et al. 1994). In the solar case, the low temperatures in the upper photosphere give rise to the cut-off frequency around 5 mHz. The response of the solar atmosphere to non-linear adiabatic shock wave propagation also leads to an appearance of the 5 mHz frequency peak in the power spectra, under the condition the underlying photosphere has a low frequency 3 mHz component (Fleck & Schmitz 1993). At the same time, Zhugzhda & Locans (1981); Gurman & Leibacher (1984); Zhugzhda (2007) argue that the observed spectrum of sunspot umbral oscillations in the chromosphere is rather due to a particular temperature gradients of the atmosphere acting as an interference filter for linear three-minute period acoustic waves. Yet another explanation was suggested by Carlsson & Stein (1997) based on the fact that at lower frequencies the energy falls off exponentially with height, stronger than for the high-frequency waves, not affected by the cut-off. Similar effect was discussed also by Bogdan & Judge (2006).

More puzzling is the horizontal distribution of wave frequencies over active regions observed at different heights. The power suppression in the umbral photosphere is thought to be due to the mode conversion extracting the energy of the incident p -mode waves (*e.g.* Cally & Bogdan 1997; Cally et al. 2003; Crouch et al. 2005, see the discussion in the next Section). The power distribution in the chromosphere can be explained assuming again the low- β slow mode propagation along the inclined magnetic field lines (Bogdan & Judge 2006; Bloomfield et al. 2007a). In the low- β plasma, the effective cut-off frequency along each individual magnetic field line decreases by a cosine of its

inclination angle θ . Taking the typical values of the inclination in the umbra and penumbra, this mechanism is able to produce the required change from the 5 mHz frequency characteristic for umbral flashes to 2-3 mHz characteristic for running penumbral waves, as confirmed by spectropolarimetric observations of Bloomfield et al. (2007a). As for the high-frequency acoustic halo surrounding active regions, no plausible explanation exists as of today. In his recently reported simulations Hanasoge (2008) calculated the rms wave power after randomly distributing acoustic sources over the region containing a sunspot-like flux tube. Both power reduction in the “umbra” and halo effect in the surrounding appeared naturally in these simulations, confirming that they are definitely an MHD effect. Certainly, the mode conversion and wave refraction in the inclined magnetic field at the edges of active regions play a role in the appearance of acoustic halo, requiring a further study.

5. Mode transformation

Mode transformation causes important effects on the observed wave propagation in active regions. The bases of the MHD wave transformation theory in a stratified solar atmosphere were developed by Zhugzhda & Dzhilov (1982). Approaching the layer where the acoustic and the Alfvén speeds are equal, the phase speeds of the different MHD modes become close and the energy can be transferred between the different branches of the dispersion relation. The direction and the effectiveness of the mode transformation depends, among the other parameters, on the wave frequency and the attacking angle between the wave vector \vec{k} and the magnetic field (*e.g.* Cally 2005, 2006; Cally & Goossens 2008). In the two-dimensional case, an approximate formula for the transformation coefficient from fast to slow mode was derived by Cally (2005) for the high-frequency waves (above the acoustic cut-off) in a vertical magnetic field:

$$T = \exp \left(-k\pi \sin^2 \psi / \left| \frac{d}{dz} \frac{c_S^2}{v_A^2} \right| \right), \quad (1)$$

where ψ is the angle between \vec{k} and the magnetic field \vec{B} . According to this equation, the fast-to-slow mode transformation is complete ($T = 1$) for the waves with \vec{k} directed along \vec{B} . For larger angles ψ the efficiency of the fast-to-slow mode transformation rapidly decreases. The higher the frequency of waves (implying generally smaller k), the smaller is the cone of ψ 's where the fast-to-slow mode transformation is effective.

Numerical simulations of the p and f mode interaction with a vertical magnetic field concentration have demonstrated that, indeed, a significant power of the incident modes is converted into the slow high- β magneto-acoustic modes propagating downwards along the magnetic field lines and leading to a visible power reduction at the surface (Cally et al. 1994; Cally & Bogdan 1997; Rosenthal & Julien 2000). In addition, these simulations have shown that the fraction of the f -mode power transformed to slow modes is sufficient to explain its power reduction in observations. On the contrary, the p modes were not transformed sufficiently in the vertical magnetic field in order to be explained by this mechanism.

Theoretical models of the fast-to-slow mode transformation in the inclined magnetic field have demonstrated that it can be particularly strong for a narrow range of the magnetic field inclinations around 20–30 degrees to the vertical (Crouch & Cally 2003; Cally 2006; Schunker & Cally 2006). Similar behavior is confirmed also in a 3D analysis by Cally & Goossens (2008). The p -mode transformation at higher frequencies is found to be significantly enhanced by moderate inclinations (Crouch & Cally 2003). The explanation of this effect is offered by the ray theory. The fast-mode high- β waves (analog of p modes) launched from their sub-photospheric lower turning points reach the transformation layer with an angle close to 20–30 degrees (Schunker & Cally 2006). Thus, for the moderately inclined magnetic field the attack angle ψ in Eq. 1 is small and the transformation is efficient. Note that, Crouch & Cally (2003); Schunker & Cally (2006); Cally & Goossens (2008) discuss the case where the slow modes produced after the transformation propagate upwards, while the downward propagation is observed in simulations by Cally et al. (1994); Cally & Bogdan (1997); Rosenthal & Julien (2000).

The critical role of the magnetic field inclination for the mode transformation is confirmed by the numerical simulations of MHD wave propagation in the chromosphere by Carlsson & Bogdan (2006), where the authors considered oscillations with wavelengths comparable to magnetic field scales. At angles smaller than 30 degrees, much of the high- β acoustic-like fast mode power is transformed into the low- β slow modes propagating along the magnetic field lines. At larger inclination angles the high- β fast modes are refracted and reflected and return back to the photosphere. Note that the larger inclinations correspond to the regions where the field strength is smaller and the transformation layer is located higher in the atmosphere. When observed at particular height, the interference pattern produced by the upward and downward propagating waves in these regions creates a ring of enhanced power around the magnetic field concentrations (Carlsson & Bogdan 2006) which can be an explanation of the acoustic halos observed at edges of active regions (Sect. 4).

6. Local helioseismology in active regions

Time-distance helioseismology makes use of wave travel times measured for wave packets traveling between various points on the solar surface through the interior (Duvall et al. 1993; Kosovichev 1999, 2002; Kosovichev et al. 2000; Zhao & Kosovichev 2003). The variations of these travel times compared to the quiet unperturbed atmosphere are assumed to be mainly due to mass flows and wave speeds below the surface. The interpretation of the inversion results of time-distance helioseismology encountered major critics when applied to magnetic active regions of the Sun. Since magnetic field modifies the wave propagation speeds in a similar way as the temperature perturbations do, the magnetic and temperature effects on the wave travel times are difficult to separate (see *e.g.* Moradi & Cally 2008). From observations, it has been demonstrated repeatedly that waves travel 20–40 seconds faster deep below the sunspots, when compared to quiet photosphere measurements (*e.g.* Duvall et al. 1993; Braun 1997). The explanation of this effect is far from clear. To understand the influence of the magnetic field on travel time measurements, the forward

modeling of waves in magnetic regions has become the preferred approach in recent years (Parchevsky & Kosovichev 2008; Hanasoge 2008; Cameron et al. 2008; Moradi & Cally 2008; Moradi et al. 2008; Khomenko et al. 2008).

Parchevsky & Kosovichev (2008) obtained that only 25% of the observed travel time difference can be explained by the magnetic field effects in their simulations of waves excited by an isolated source in an inclined magnetic field. Other authors report values of the travel time differences from simulations that are similar to the observed ones (Cameron et al. 2008; Moradi & Cally 2008; Moradi et al. 2008; Khomenko et al. 2008). The dependence of the magnitude of the travel time difference on the magnetic field strength was shown by Cameron et al. (2008) and Khomenko et al. (2008), while Moradi et al. (2008) argue travel time differences are largely insensitive to the sunspot structure. The frequency dependence of the travel time measurements reported in the literature (Couvidat et al. 2006; Couvidat & Rajaguru 2007; Rajaguru 2008; Braun & Birch 2008) is interpreted as one of the demonstrations of the surface magnetic field effects. High frequency waves have, in general, larger travel-time differences (Rajaguru 2008). Qualitatively, such dependence is reproduced by numerical simulations as well as the ray theory eikonal approach (Moradi et al. 2008; Khomenko et al. 2008). The overall agreement between the simulations and observations indicates that the observed time-distance helioseismology signals in sunspot regions correspond to fast high- β MHD waves (modified p and f modes). This conclusion apparently contradicts the chromospheric measurements, where the correlations between the photospheric and chromospheric signals was found and the longitudinal propagation of the low- β slow waves allowed the best agreement with the observations (Centeno et al. 2006; Bloomfield et al. 2007a). In the same vein, Schunker et al. (2005); Schunker & Cally (2006) argue that the observed dependence of the Doppler signal on the line-of-sight magnetic field inclination angle requires an alignment between the magnetic field direction and the wave propagation, only possible for low- β slow waves. More work is needed to clarify these issues including the calculation of the line formation heights and sunspot Wilson depression effect.

7. Conclusions

Theoretical interpretation of waves observed in sunspots has made big advances during the last decades. The influence of the $\beta = 1$ level and the magnetic field inclination on the observed wave properties is being clarified, allowing the interpretation of the observed oscillations in term of MHD wave modes. The observed wave pattern in the upper photosphere and the chromosphere is found to be compatible with the low- β acoustic-like slow mode propagation along the inclined magnetic field lines. Several physical effects are proposed to be responsible for the frequency change with height in the umbra from the 3 mHz in the photosphere to 5–6 mHz in the chromosphere. The mode transformation at the $\beta = 1$ level is proposed to be responsible (at least in part) for the wave power distribution over the active regions, including the wave suppression in the umbra and, possibly, enhanced power of acoustic halos. Theoretical interpretation of the local helioseismology measurements including magnetic field has been initiated. The first simulations of helioseismic waves in sunspots show a general

agreement of the wave travel times of the high- β fast modes with observations. Still, the final issue in this field is far from clear and the major developments are expected in the future. The 3-dimensional wave propagation and mode transformation in sunspots are important to analyze from simulations, including the chromospheric layers.

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